

# Experimental realization of any discrete operator

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## Abstract

A straightforward argument shows that, by allowing counterfactual elements of physical reality, any *arbitrary* discrete finite-dimensional operator corresponds to an observable.

As can be readily verified, any matrix  $A$  can be decomposed into two self-adjoint components  $A_1, A_2$  as follows.

$$A = A_1 + iA_2 \tag{1}$$

$$A_1 = \frac{1}{2}(A + A^\dagger) =: \Re A, \tag{2}$$

$$A_2 = -\frac{i}{2}(A - A^\dagger) =: \Im A. \tag{3}$$

This is an extension of the decomposition of a complex number into its real and imaginary component; e.g., for any  $a \in \mathbb{C}$ ,  $a = a_1 + ia_2$ , with  $a_1 = (1/2)(a + a^*) =: \Re a \in \mathbb{R}$  and  $a_2 = -(i/2)(a - a^*) =: \Im a \in \mathbb{R}$  if  $A = a$  is a  $(1 \times 1)$ -matrix. Since the trace is additive, this decomposition preserves the usual definition of quantum expectation values even for the case of non pure states.

Thus, if  $A_1$  and  $A_2$  would consistently be measurable, then  $A$  would consistently be measurable. As could be expected, in quantum mechanics, this is guaranteed only if their commutator vanishes; i.e.,

$$[A_1, A_2] = A_1A_2 - A_2A_1 = 0.$$

Note that, if  $A = A^\dagger$  is self-adjoint, the decomposition is trivial; i.e.,  $A_1 = A$  and  $A_2 = 0$ .

Nevertheless, quantum mechanics could be extended to counterfactual elements of physical reality, as suggested by the Einstein-Podolski-Rosen (EPR) argument [3]. EPR state that “if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.” Thereby, EPR make no difference between an observable which is actually measured and one which could only be obtained by reasoning. The term *counterfactual* can be defined, in Max Jammer’s words [5, pp. 9–10], as follows. “In general, an argument is called “counterfactual” if it involves a thought experiment the actual performance of which on a given system is made impossible because the conditions necessary for performing this experiment cannot be satisfied.” A shorter definition is due to Roger Penrose. [9, p. 204], “... things that might have happened, although they did not happen.” Surely enough, as has been shown by Bell [1] (with an explicit reference to Gleason’s theorem [4]) and by Kochen and Specker [6], the naive assumption of non contextual counterfactuals gives rise to inconsistencies and therefore excludes a broad class of hidden parameter theories [10, 7].

With this *proviso*, one could nevertheless imagine Gedankenexperiments which “measure” an arbitrary discrete observable, irrespective of whether the corresponding operator is self-adjoint or not.—Here, the quotes surrounding “measure” mean that counterfactuals are involved. Assume, for instance, a spin one-half system representable by twodimensional Hilbert space. The matrix

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

is not self-adjoint, but following equations (1)–(3) can be decomposed into two self-adjoint operators

$$A_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}\sigma_1, \quad (4)$$

$$A_2 = \frac{1}{2} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = -\frac{1}{2}\sigma_2, \quad (5)$$

$$A = \sigma_1 - i\sigma_2. \quad (6)$$

Here,  $\sigma_i, i = 1, 2, 3$  denote the Pauli spin matrices. In an EPR-type measurement setup drawn in Figure 1, two entangled spin one-half particles in the singlet state could be spatially separated and interrogated. Measurement on the first particle could yield the element of physical reality associated with  $\sigma_1$ ; that is the spin state along the  $x$  – *axis*. Measurement on the second particle could yield the element of physical reality associated with  $\sigma_2$ ; that is the spin state along the  $y$  – *axis*. These outcomes could then be combined according to Equation (1) to yield the element of physical reality associated with  $A$ .

It is not difficult to imagine that, since we can always decompose an arbitrary observable into just two self-adjoint observables, a generalized EPR-type

experiment combined with the Reck–Zeilinger–Bernstein–Bertani setup [11], every discrete finite-dimensional operator which is not necessarily self-adjoint, is “measurable” (in the counterfactual sense).

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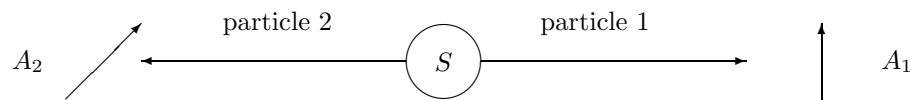


Figure 1: EPR-type setup for measurement of arbitrary operator decomposed into its self-adjoint real and imaginary part.